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VELOCITY PROFILES AND FORM OF A LAMINAR JET IN IMMISCIBLE LIQUID–LIQUID SYSTEMS

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Abstract—An exact numerical solution for the velocity profiles and the form of a laminar jet in immiscible Newtonian liquid–liquid systems is obtained. The main difficulty connected with the simultaneous integration of the equations of motion of the jet and of the continuous phase in the presence of an unknown interface is overcome by using an indivisible finite-difference scheme. This avoids any additional iteration for defining the unknown velocities of the points of the interface. The solution of the equations of motion as written in a boundary layer approximation is a function of the following non-dimensional numbers: the Reynolds numbers of each phase, the Weber and Froude numbers and the ratio of the densities. The influence of some of these parameters on the jet behaviour is illustrated as well as the influence of the initial velocity profiles at the nozzle exit.

A comparison is made with some known results. Important differences are found to exist between the exact and the approximate velocity profiles and their gradients at the interface. It seems that these differences result from the comparatively inexact description of the boundary layer of the continuous phase when using moment methods. Such a conclusion limits the applicability of the approximate moment solutions to heat and mass transfer problems as well as to the jet stability analysis.

1. INTRODUCTION

The velocity profiles and the jet radius of a laminar jet form the basis for investigation of numerous mass- and heat-transfer problems in liquid–liquid systems. These quantities are also necessary for the analysis of the jet instability which results from the propagation of small disturbances over the interface. Investigation of these problems is of interest for determination of the interfacial area of a liquid–liquid extraction column.

The first numerical solution of the problem of defining the velocity profile and the contraction of a liquid laminar jet belongs to Duda & Vrentas (1967). These authors have considered a jet running into a gaseous continuous phase and accounted for the presence of the latter by only introducing a pressure term into the boundary condition at the jet surface. In order to overcome the main difficulty resulting from the presence of a free boundary and to apply a finite-difference scheme, Duda & Vrentas have introduced Protean coordinates, i.e. the stream function together with the axial coordinate have been chosen for new independent variables. The analysis of Duda & Vrentas needs to be extended to the case when both the dispersed and the continuous phases are viscous liquids. Such an extension is necessary when the effects of the density and viscosity of the continuous phase are of interest.

Within the formulation of Yu & Scheele (1975) this more complicated problem has been reduced to the integration of a simultaneous system of nonlinear partial differential equations. To the difficulties connected with the existence of an interfacial free boundary has been added a new one, caused by the necessity of the simultaneous integration of the equations of motion of the continuous and dispersed phases. Yu & Scheele have substituted for the exact solution an approximate one constructed by means of a moment integral method. Various moment methods have been also used in other papers (see Meister & Scheele 1969; Penchev *et al.* 1977, etc.).

In this paper the application of the finite-difference method has been extended to the general case of a jet in liquid–liquid systems. A direct numerical solution of the equations of motion as stated by Yu & Scheele (1975) has been obtained. In the framework of the proposed method the Duda & Vrentas solution appears as a particular case.

The necessity of establishing a direct numerical method arises naturally since the above mentioned approximate solutions are numerical. The main requirement for such a method is that the computational time be in acceptable limits.

The main point of the proposed method is the finite-difference formulation of the indivisible boundary problem for both phases avoiding any additional iteration for the unknown velocities of the points of the interface. This makes the computational time comparable to the average time necessary for an approximate numerical solution.

2. EQUATIONS OF MOTION

Consider a vertical stationary axisymmetric liquid jet running with an average axial velocity U_N into another liquid out of a nozzle of a circular cross section of diameter D_N. Suppose that both the jet and the surrounding liquid are incompressible, Newtonian and immiscible and that the fluid system thus obtained lacks heat- and mass-transfer. The geometrical configuration of the flow is shown in figure 1.

Because of the flow symmetry a cylindrical coordinate system is applied with axis z coincident with the jet axis (in the direction of the injection) and origin at the centre of the nozzle exit.

The equations of motion and the boundary conditions are reduced to dimensionless form with the characteristic scale parameters D_N and U_N. Variables with subscripts 1 and 2 refer to the continuous and dispersed phase respectively.

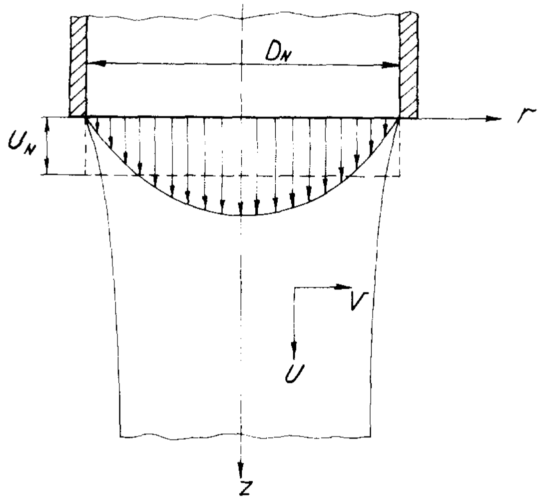

Figure 1. Flow configuration.

(a) *Continuous phase*

The complete set of equations of motion of the continuous phase includes two dimensionless parameters—Reynolds number $Re_1 = (D_N \rho_1 U_N)/\mu_1$ (where ρ_1 and μ_1 are correspondingly density and dynamic viscosity) and Froude number $Fr = U_N^2/(D_N g)$ (where g is acceleration of gravity).

The analysis of the orders of magnitude (Yu & Scheele 1975) shows that when $Re_1 > 100$ the simplification of the boundary layer theory may be applied to the equations of motion. The boundary layer equations concerning the continuous phase when written in cylindrical coordinates in dimensionless form read

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_1) + \frac{\partial U_1}{\partial z} = 0, \tag{1}$$

$$V_1 \frac{\partial U_1}{\partial r} + U_1 \frac{\partial U_1}{\partial z} = -\frac{\rho_2}{\rho_1}\frac{\partial P_1}{\partial z} + \frac{2}{Re_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_1}{\partial r}\right)\right] \pm \frac{1}{2Fr}, \tag{2}$$

$$\frac{\partial P_1}{\partial r} = 0, \tag{3}$$

where (V, V) are the velocity components.

Signs (+) and (−) in front of the term $1/(2Fr)$ in [2] correspond to a jet injected downwards and upwards respectively. Thus for a jet injected upwards [1]–[3] are identical to those of Yu & Scheele (1975).

Only the first case (that is a jet injected downwards) will be considered for reasons of definiteness. Equation [3] represents a well known result of the boundary layer theory, i.e. the independence of the pressure on the radial coordinate of the layer. With this in mind and because of the immobility of the continuous phase out of the layer, i.e.

$$\frac{\rho_2}{\rho_1}\frac{\partial P_1}{\partial z} = \frac{1}{2Fr}, \qquad [4]$$

one can exclude the pressure gradient term from [2]:

$$V_1\frac{\partial U_1}{\partial r} + U_1\frac{\partial U_1}{\partial z} = \frac{2}{Re_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_1}{\partial r}\right)\right]. \qquad [5]$$

(b) *Dispersed phase*

One can obtain the equations of motion of the jet liquid by substituting subscript 2 for the subscript 1 in [1]–[3].

The analysis of the orders of magnitude of the equations of motion of the dispersed phase belongs to Duda & Vrentas (1967) and Markova & Shkadov (1972). The main mathematical difficulties in solving the equations of motion of both phases arise from the existence of the unknown interface (the jet surface).

To overcome these difficulties Duda & Vrentas (1967) have applied Protean coordinates (ψ, ζ) by means of which the phase surface equation reads $\psi = \text{const} = C_\psi$.

The existence of C_ψ actually results from the condition of mass conservation within the jet. In terms of the chosen dimensionless parameters and upon the condition that $\psi = 0$ at $r = 0$, $C_\psi = 1/2$. In what follows the equations of motion of the dispersed phase will be used as derived in detail by Duda & Vrentas (1967), i.e.

$$\frac{\partial \varphi_2}{\partial \zeta} = -\frac{2\partial P_2}{\partial \zeta} + 4\left[\frac{\partial \varphi_2}{\partial \psi} + \frac{r^2\varphi_2^{1/2}}{2}\cdot\frac{\partial^2\varphi_2}{\partial \psi^2}\right] + \frac{Re_2}{Fr}, \qquad [6]$$

$$\frac{\partial P_2}{\partial \psi} = 0, \qquad [7]$$

where $\zeta = z/Re_2$ and $\varphi_2 = U_2^2$ are introduced for convenience.

In [6] the radial coordinate r of an arbitrary point of the jet is supposed to be a function of ψ and ζ, defined as

$$\frac{\partial r^2}{\partial \psi} = \frac{2}{\varphi_2^{1/2}}. \qquad [8]$$

Equations [6]–[8] form a set of nonlinear partial differential equations which describe the distribution of the axial velocity within the jet. For defining the radial component one should use the equation

$$V_2 \cdot Re_2 = \varphi_2^{1/2}\frac{\partial r}{\partial \zeta}. \qquad [9]$$

The method of elimination of the term $\partial P_1/\partial z$ from [2] could be also applied to the term $\partial P_2/\partial \zeta$ in [6]. The condition of continuity of the pressures P_1 and P_2 through the interface which is necessary for such an elimination contains an extra term caused by the interfacial tension. The elimination concerning [6] will be presented in the next section.

Protean coordinates are mostly suitable for the case of a gaseous continuous phase which acts on the jet through the pressure P_1 only (Duda & Vrentas 1967).

In the considered case of a liquid–liquid fluid system these coordinates are more suitable than the cylindrical coordinates when the flow within the jet is concerned. Cylindrical coordinates (r, ζ) are preferable in the region of the outer boundary layer because they lead to a simpler form of the boundary conditions at the outer boundary. Note that this boundary is not a streamline. Thus in Protean coordinates, the boundary is described by the equation $\psi = \psi(\zeta)$, where $\psi(\zeta)$ is an unknown function which lacks prior constructive arguments.

Equations [1] and [5] are written for convenience in terms of the new variables ζ and $\varphi_1 = U_1^2$, that is

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_1) + \frac{1}{Re_2}\frac{\partial \varphi_1^{1/2}}{\partial \zeta} = 0, \qquad [10]$$

$$V_1\varphi_1\frac{\partial \varphi_1}{\partial r} + \frac{\varphi_1^{3/2}}{Re_2}\frac{\partial \varphi_1}{\partial \zeta} = \frac{4\varphi_1^{3/2}}{Re_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{2\varphi_1^{1/2}}\frac{\partial \varphi_1}{\partial r}\right)\right]. \qquad [11]$$

(c) *Initial velocity profiles*

Using these profiles one prescribes the flow of the two phases in the initial cross section ($\zeta = 0$). In the proposed solution profiles of general type may be used, i.e.

$$\varphi_1(r) = g_1(r), \qquad [12]$$

$$\varphi_2(\psi) = g_2(\psi), \qquad [13]$$

where $g_1(r)$ and $g_2(\psi)$ are theoretically or experimentally defined functions.

Two versions (as mostly used) of condition [13] will be considered here, namely

$$g_2(\psi) = 4(1 - 2\psi), \qquad [14]$$

$$g_2(\psi) = 1. \qquad [15]$$

The fully developed parabolic profile [14] corresponds to a jet issuing out of a sufficiently long nozzle. Later an initial profile of the form

$$g_1(r) \equiv 0 \qquad [16]$$

will be considered which corresponds to a nozzle ending in a hole in an infinite plate. Equation [15] together with [16] correspond to a jet issuing out of a hole in an infinite thin plate. Both [14] and [15] profiles satisfy the mass balance condition with $C_\psi = 1/2$. Note that one can use another profile (defined theoretically or experimentally) instead of [16]. Then the general case of a jet issuing out of a nozzle into a movable or immovable continuous phase could be also analysed. When (as in the considered case) the nozzle exit is surrounded by a plate the initial profile of the continuous phase is known in advance.

3. BOUNDARY CONDITIONS

The equations of motion of both phases should be completed with boundary conditions describing the flow along the jet centerline and on the outer surface of the boundary layer of the continuous phase. Conditions describing the interaction between the jet flow and the surrounding liquid through the interface should be also imposed.

(a) *Interface* ($\psi = 1/2$)

As has been shown by Yu & Scheele (1975) the conditions of continuity of the shear and normal stresses through the interface can be reduced to the form:

$$\mu_1 \frac{\partial U_1}{\partial \psi} = \mu_2 \frac{\partial U_2}{\partial \psi}, \qquad [17]$$

$$P_2 - P_1 = \frac{2}{R \cdot We}, \qquad [18]$$

where $R(\zeta)$ is the unknown radius of the jet and $We = (D_N U_N^2 \rho_2)/T$. The pressure jump as expressed in the right side of [18] is caused by an interfacial tension T. Note that the pressures P_1 and P_2 in [2], [6] and [18] are made dimensionless with respect to $\rho_2 U_N^2$.

With the aid of [18], [6] can be written in a simpler form (as it was underlined above). Solving [18] for $\partial P_2/\partial \zeta$ and taking into account [4] (for $\partial P_1/\partial \zeta$) one determines the equation

$$\frac{\partial \varphi_2}{\partial \zeta} = N_j + \frac{2}{We(R^2)^{3/2}} \frac{dR^2}{d\zeta} + 4\left[\frac{\partial \varphi_2}{\partial \psi} + \frac{r^2 \varphi_2^{1/2}}{2} \frac{\partial^2 \varphi_2}{\partial \psi^2}\right] \qquad [19]$$

instead of [6], where

$$N_j = \frac{Re_2}{Fr}\left(1 - \frac{\rho_1}{\rho_2}\right).$$

The factor $1 - (\rho_1/\rho_2)$ corresponds to a jet injected downwards. The continuity conditions for the radial and axial velocity components must be satisfied, i.e.

$$U_1 = U_2 \qquad (\varphi_1 = \varphi_2), \qquad [20]$$

$$V_1 = V_2. \qquad [21]$$

Equation [20] expresses the nonslip condition of the liquids along the interface and together with [17] describes the interaction between the phases. Equation [21] describes the standard condition of absence of mass flow through the interface. With the aid of [20] condition [17] may be reduced to the following simpler form

$$\mu_1 \frac{\partial \varphi_1}{\partial r} = \mu_2 R \varphi_2^{1/2} \frac{\partial \varphi_2}{\partial \psi}. \qquad [22]$$

(b) *Jet axis* ($\psi = 0$)

In the above considerations a condition $\psi = 0$ at $r = 0$ was used. Since ψ is now an independent variable this condition will be used then in the form

$$r^2 = 0 \qquad [23]$$

at $\psi = 0$.

Equation [23] applied to the points of the jet axis leads to a new boundary condition. Formally this condition follows from [19] with the function r^2 supposed to be equal to zero,

$$\frac{\partial \varphi_2}{\partial \psi} = \frac{1}{4}\left[\frac{\partial \varphi_2}{\partial \zeta} - N_j - \frac{2}{We(R^2)^{3/2}} \frac{dR^2}{d\zeta}\right]. \qquad [24]$$

Conditions [23] and [24] are identical to those of Duda & Vrentas (1967).

(c) *Outer edge of the boundary layer*

Formally the boundary condition must be written as follows

$$U_1 = 0 \qquad (\varphi_1 = 0). \qquad [25]$$

This corresponds to a continuous phase which is immovable out of the boundary layer. Strictly speaking [25] is valid at $r \to \infty$. But it is known from the boundary layer theory that this can be satisfied up to a sufficient degree of accuracy at $r = r_{max}$, where r_{max} is generally a function of ζ. Later this function is defined from numerical experiments.

4. DIRECT NUMERICAL SOLUTION

The equations of motion of the continuous ([10] and [11]) and dispersed ([8], [9] and [19]) phases together with the corresponding boundary ([25] and [23], [24]), initial ([16] and [14] or [15]), and interfacial ([20], [21], [22]) conditions form a set of nonlinear partial differential equations which describe the velocity distribution. In what follows a finite-difference method is applied for solving this set of equations.

The equations of motion were first approximated in the sense of the finite-difference method over a suitable grid, shown in figure 2. The scheme is of the second order of approximation and is implicit in the ζ direction. The grid was constructed in such a way that its lines with a step $\Delta\psi$ along the axis r inside the jet $0 \le \psi \le 1/2$ coincide with the streamlines. When the outer layer region is considered these lines with a step Δr are equidistant starting with the line $\psi = 1/2$. The grid lines along ζ with a step $\Delta\zeta$ are parallel to the axis r. The values of $\Delta\psi$ and Δr are defined in accordance with the required accuracy of the numerical solution.

An outer iterative procedure was constructed for determining the unknown radius of the jet. If in the process of the solution an approximation of the jet radius has been obtained then the interface position and the entire flow geometry would be determined.

The task is to find the corresponding velocity distribution after the flow geometry is determined. An inner iterative procedure is used for that purpose because of the nonlinearity of the system of algebraic equations obtained from the approximation of the nonlinear system of partial differential equations.

After a standard linearization, the finite-difference versions of [19] and [11] for the dispersed and continuous phase are

$$A_i^{(k-1)}\varphi_{i-1,j}^{(k)} + C_i^{(k-1)}\varphi_{i,j}^{(k)} + B_i^{(k-1)}\varphi_{i+1,j}^{(k)} = F_i^{(k-1)}, \qquad [26]$$

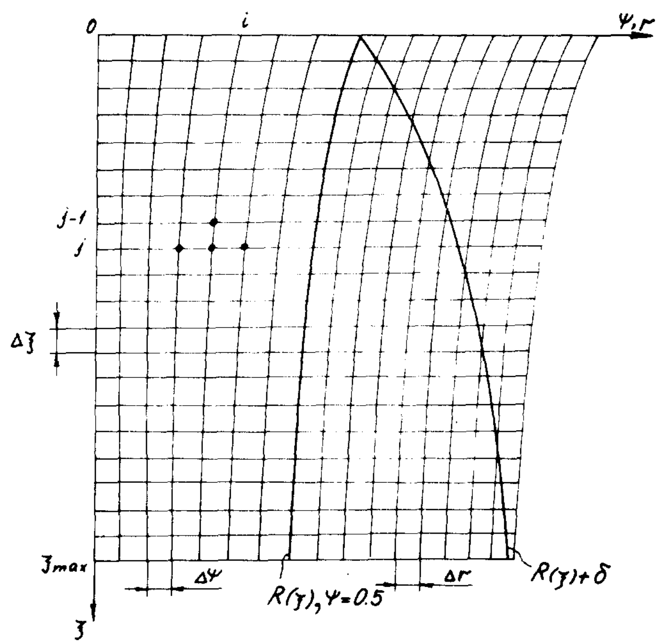

Figure 2. Grid of the finite-difference scheme.

where k is the number of the internal iterations. The coefficients A_i, B_i, C_i and F_i are known functions of φ, r^2 and V at the knots of the row analysed in the course of the $k-1$ iteration. F_i is at the same time a function of the values of φ on the previous row. Subscripts 1 and 2 in [26] for the continuous and dispersed phase have been omitted.

The finite-difference form of [22] at the interface point of the j-row is analogous to [26]. For the first two points of the row [24] is added to [26] and for the last two points of the same row [25] is added. Thus the matrix of the set [26] is tridiagonal and its solution may be found by an elimination method.

The procedure described yields an indivisible boundary problem for both the jet and continuous phase liquids. In such a way any additional iterative procedure for the unknown velocities of the points of the interface is eliminated.

The k-approximation of r^2, V and φ for every point of the row analysed is determined with the aid of the k-approximation of the axial velocity distribution. In this way the k-approximation of the j-row is completed.

The results obtained are used for calculating the new values of the coefficients A_i, B_i, C_i and F_i. Then the inner iteration is repeated until the prescribed initial accuracy is reached. The velocity distribution calculated for the j-row is used as a zero approximation for the next $j+1$ row ($j>1$). Equations [14] or [15] and [16] yield a value of φ which is used for similar calculations concerning the first row ($j=1$).

After passing through all the rows along ζ-line one automatically obtains the next approximation for the jet radius $R(\zeta)$ by using the solution of [8] with $\psi = 1/2$. The outer iterative procedure described above ends after reaching the previously given accuracy. In this way the final numerical solution of the problem is obtained. The zero approximation for $R(\zeta)$ is found by the method of Targ-Shvetz (see Penchev *et al.* (1977)) or by means of a function of the type $R(\zeta) = 1 - \exp(-C\zeta)$ (C is a suitable constant). The iterative process is numerically stable and converges to the exact solution. The rate of convergence does not practically depend on the type of the zero approximation for $R(\zeta)$. A program in Fortran IV was written. The average time for a given version of a numerical solution (up to an accuracy of 10^{-3} with respect to R) does not exceed 8 min on IBM 360/45.

5. ANALYSIS OF THE NUMERICAL RESULTS

(a) *Computations*

The calculated cases are described in table 1 which contains chosen values of all of the five characteristic dimensionless numbers. Case 1 is considered as a basic one. It corresponds to a real liquid–liquid system (methane/tetrachloride–water). The Reynolds numbers of the continuous and dispersed phases were obtained from experimentally observed velocities of injection which guarantee laminarity of the jet flow and validity of the boundary layer approximation. Cases 1–3 illustrate the effects of the ratio μ_2/μ_1 and the continuous phase Reynolds number Re_1 on the jet contraction and velocity distribution. The values of the dimensionless numbers in cases 4–8 are identical to those of Yu & Scheele (1975), so that a comparison between the two types of solution can be made. In all the cases 1–8 a parabolic

Table 1.

Case	N_J	We	Re_2	Re_1	μ_2/μ_1	ρ_2/ρ_1	Fr
1	121.47	2.81	660.47	400	0.969	1.600	2.039
2	121.47	2.81	660.47	200	0.969	3.200	3.738
3	121.47	2.81	660.47	400	2	3.302	3.790
4	128.46	5.4	835	1670	1	0.500	6.500
5	128.46	5.4	835	1670	1	0.500	6.500
6	128.46	5.4	835	—			6.500
7	128.46	54	835	1670	1	0.500	6.500
8	128.46	54	835	1670	1	0.500	6.500

initial axial velocity profile was used except in cases 5 and 8, where a flat one was adopted. Cases 4–8 correspond to a jet injected upwards (see table 1). The calculations were performed with the following steps:

$$\Delta\psi = 0.0025 \quad \text{along } \psi \text{ line}$$

$$\Delta r = 0.00125 \quad \text{along } r \text{ line}$$

$$\Delta\zeta = 0.005 \quad \text{along } \zeta \text{ line.}$$

The prescribed accuracy was 10^{-4} with respect to the internal iterations and 10^{-3} with respect to $R(\zeta)$. The results obtained differ from each other slightly when doubling $\Delta\zeta$, $\Delta\psi$, Δr and this practically confirms the stability of the numerical method.

As for the choice of the r_{max} value in [25] it is conventional practice to balance between the accuracy and the economy of the computational time. Numerical experiments were performed with r_{max} and $2r_{max}$ for different values of Re_2 numbers. Doubling chosen values of r_{max} did not practically affect the numerical results.

(b) *Velocity distributions*

Both the jet and the continuous phase stream lines in the case 1 are shown in figure 3. As is seen the stream lines inside the jet are almost straight. Some deviations are observed in the region near the nozzle. The line $\psi = 1/2$, corresponding to the interface, becomes curved near the nozzle, which is caused by the sudden acceleration of the jet surface in that region. At sufficient distances from the nozzle (5–8 $D_N/2$) this line is almost straight. So the jet surface can be approximately considered as a conic one (see also Skelland & Huang 1977). The continuous phase streamlines near the nozzle are almost perpendicular to the outer edge of the boundary layer presented by the line $U_1 = 1\%\ U_{2max}$ in figure 3.

At sufficient distances from the nozzle the lines $\psi = \text{const}$ are almost parallel to the interface.

In figure 4 the axial velocity distribution for three different cross sections of the jet is drawn. Downstream the axial velocities of the points of the jet axis and surface increase. The velocity profile relaxation according to Yu & Scheele (1975) is caused by the jet momentum exchange which results from the acceleration of gravity, the surface tension and the viscous interaction between the phases. The axial velocity gradient on the interface increases as well.

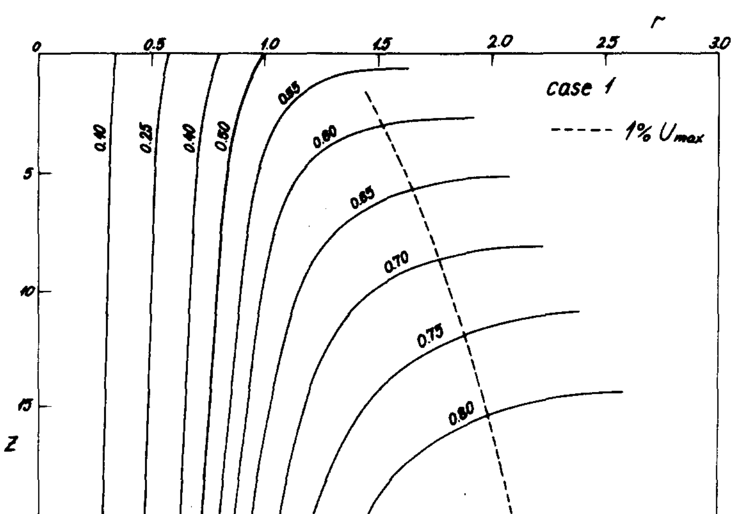

Figure 3. Stream lines.

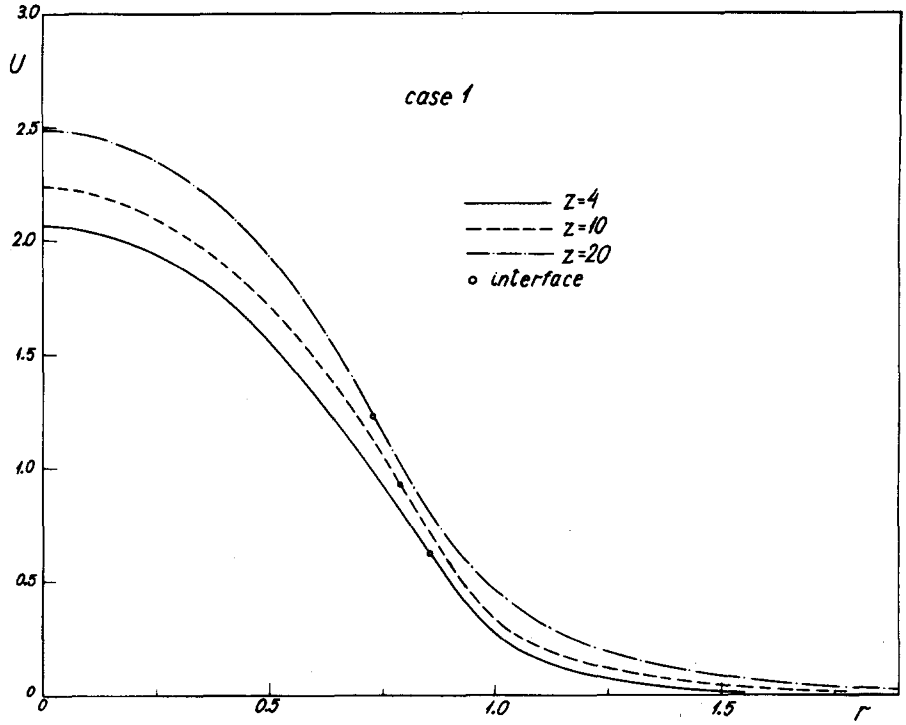

Figure 4. Axial velocity distribution.

A uniform axial velocity distribution is obtained when a nonviscous outer phase is considered (Duda & Vrentas 1967; Markova & Shkadov 1972).

When a viscous continuous phase is considered the existence of a shear stress on the interface does not allow relaxation of the axial velocity profile into a flat one. The radial velocity modulus distribution is shown in figure 5. It increases near the nozzle exit and reaches

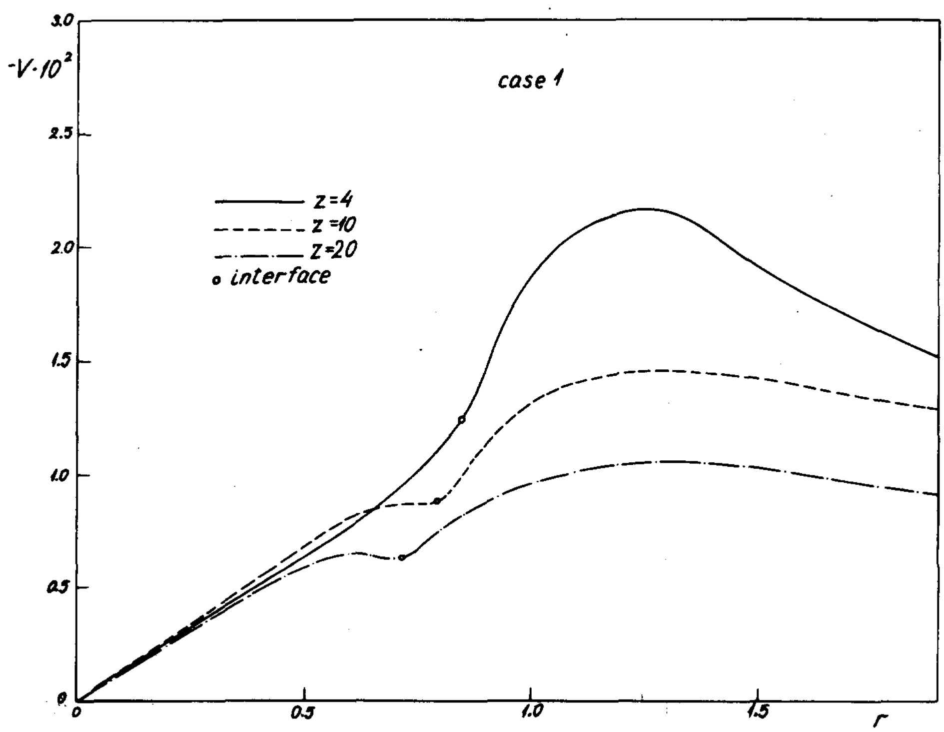

Figure 5. Radial velocity distribution.

its maximum value within the outer boundary layer. Downstream the radial velocity modulus reaches two maxima; the first—inside the jet and second—inside the layer. The radial component of the velocity is negative everywhere.

(c) *The effect of initial velocity profiles*

It was noted that parabolic and flat profiles were used for the initial axial velocity distribution [14], [15]. The jet radius contractions are compared in figure 6, obtained from [14] and [15]—cases 4 and 5 respectively. In the region near the nozzle the jet contraction corresponding to a flat initial profile (case 5) is weaker than in the case of an initial parabolic profile (case 4). The opposite situation is observed for the shear stress τ, figure 8. The surface axial velocity variation in the case 5 on the same figure shows that the axial velocity profile relaxation is weaker. The downstream U_s and τ variations in the cases 4 and 5 are almost identical.

(d) *Comparison between the results of the direct numerical and the approximate solutions*

Figures 6 and 7 show correspondingly the jet contraction and the axial velocity distribution as obtained from the proposed direct numerical solution and compared to the jet contraction and the velocity distribution obtained by Yu & Scheele (1975).

In figure 6 the jet radii as obtained from different methods are presented. In particular the Duda & Vrentas (1967) result is drawn. The initial velocity profile is parabolic in all the cases. Note that a comparison with the work of Yu & Scheele (1975) is not possible in the case of a flat initial profile because their method does not include this case.

The jet radius coincidence with the two methods taken into account is sufficiently accurate—figure 6. The results of the proposed method in the case of a nonviscous continuous phase are practically identical to those of Duda & Vrentas (1967). When compared to Yu & Scheele (1975) results, the jet radii are in good agreement. But the comparison of the velocity profiles for viscous continuous phase (figure 8) shows considerable difference. The axial velocity gradients on the interface (τ/μ_2) differ from each other by more than 10%, while the surface velocity values differ by more than 20%. The agreement between the axial velocity profiles is much weaker for the continuous than the dispersed phase, possibly because of the

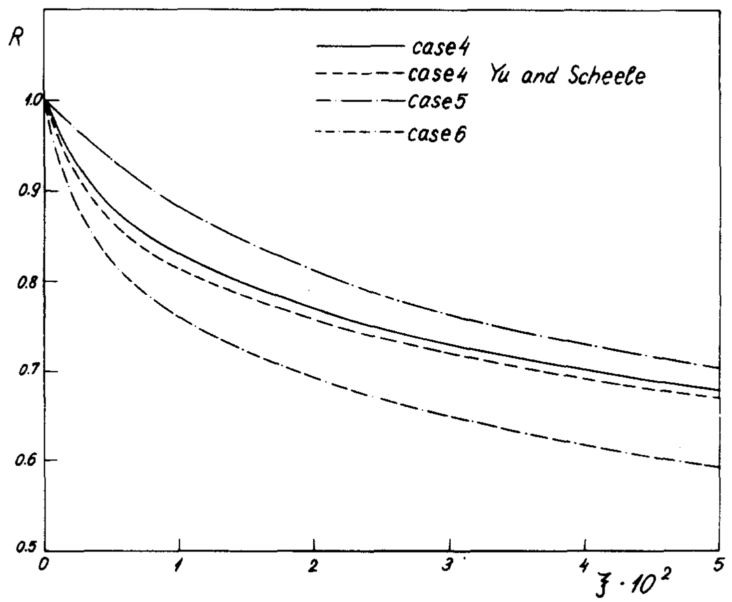

Figure 6. Comparison between the exact and approximate jet radii.

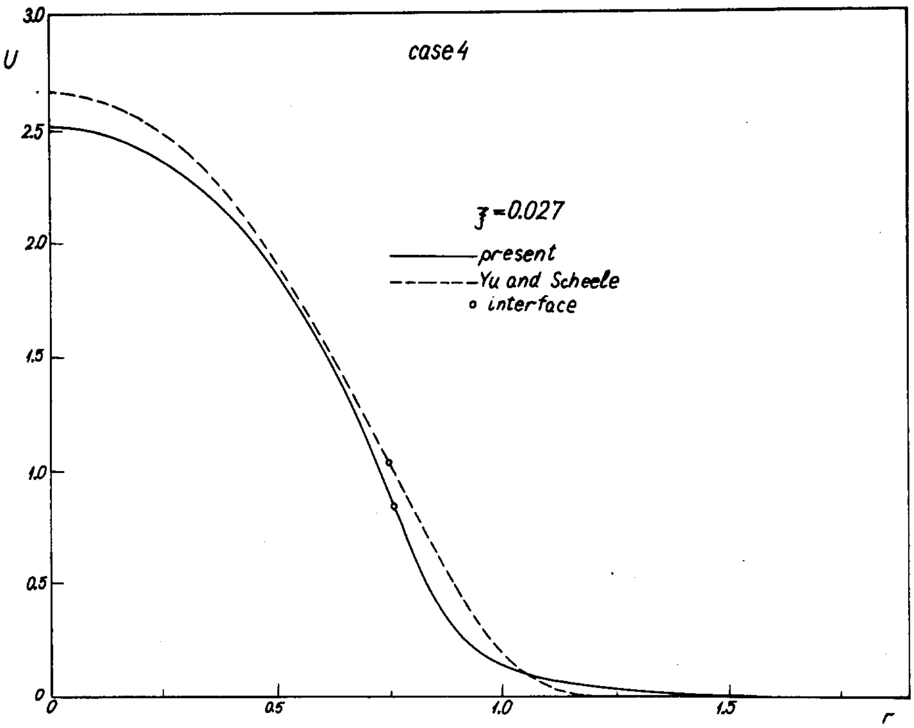

Figure 7. Effects of the initial profiles on the surface velocity and shear stress.

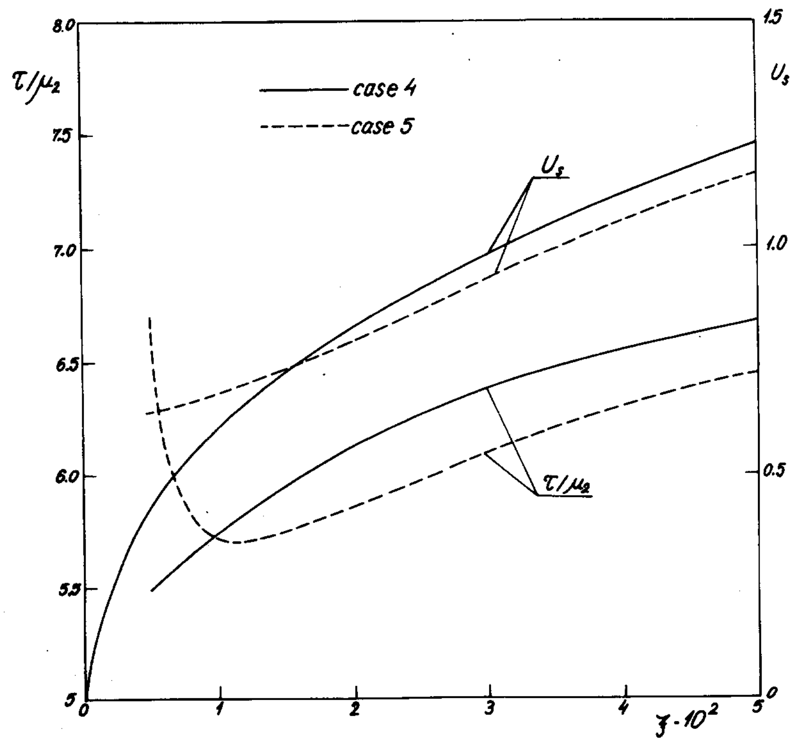

Figure 8. Comparison between the exact and approximate axial velocity distributions.

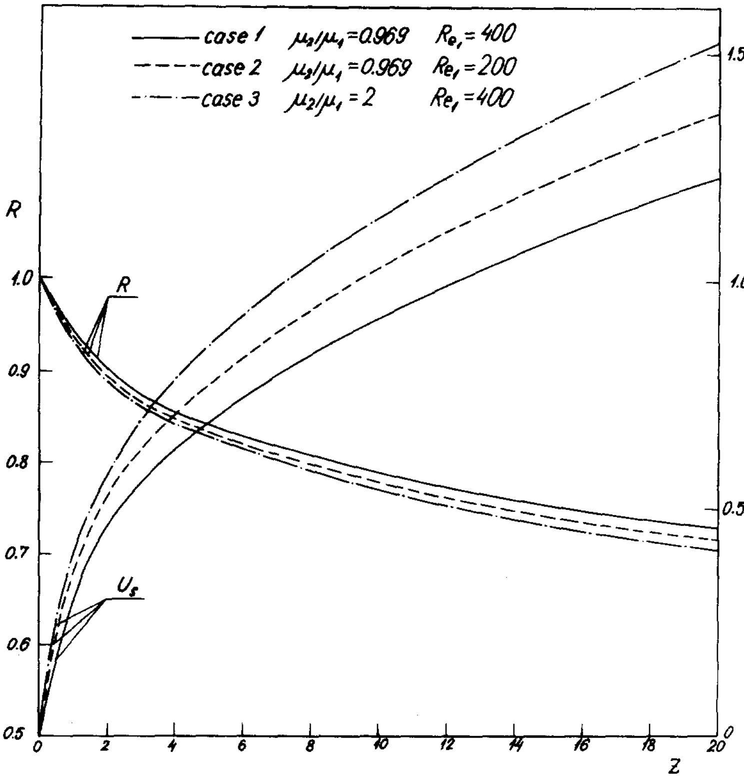

Figure 9. Effect of μ_2/μ_1 and Re_1 on the jet radius and surface velocity.

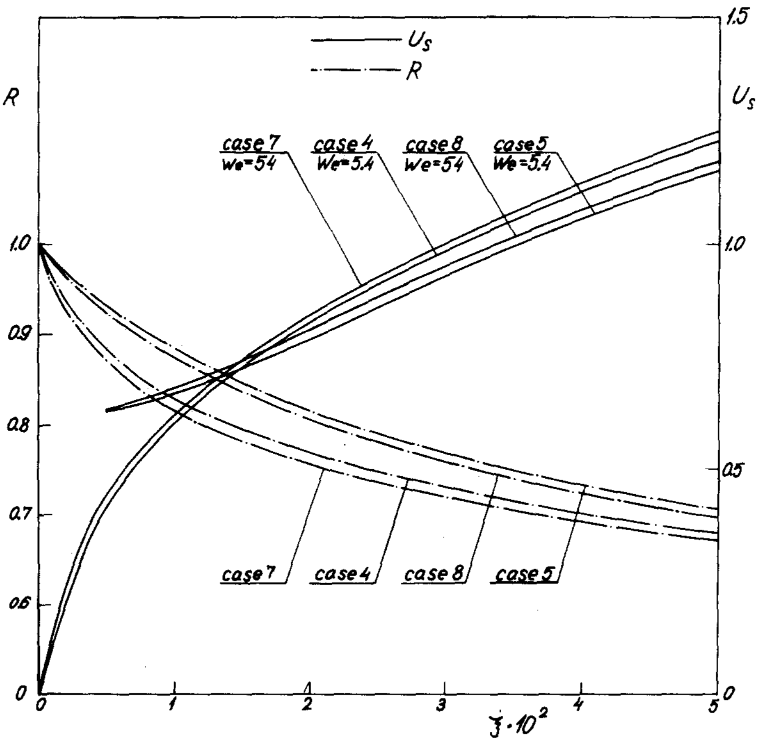

Figure 10. Effect of We and the initial profile on the jet radius and surface velocity.

inherent inaccuracy of the integral methods of the boundary layer theory noted by Yu & Scheele (1975).

(e) *The effect of the viscous ratio μ_2/μ_1, continuous phase Reynolds number Re_1 and Weber number We*

The influence of the variation of μ_2/μ_1 and Re_1 (cases 2 and 3) on the jet contraction and surface velocity U_s is presented in figure 9. All of the other injection conditions are identical to those of case 1. The effect of increasing of the ratio μ_2/μ_1 from the value 0.969 up to 2 is greater than the double decreasing of Re_1. The variations of Re_1 and μ_2/μ_1 influence considerably the axial velocity profile relaxation while the same variations slightly affect the jet radius.

The influence of Weber number on the surface velocity and jet radius $R(\zeta)$ for different initial profiles is shown in figure 10.

The effect of We 10 time increasing on the jet contraction and surface velocity variation is much weaker than the effect of substituting a flat initial profile for the parabolic one. This supports the assumption of Yu & Scheele (1975) that the jet relaxation near the nozzle depends considerably on the type of the initial velocity profile.

CONCLUSIONS

The problem of determining the form and the velocity profiles of a jet flowing into a viscous continuous phase has been solved by means of a direct numerical solution. Two different initial profiles have been used for describing the distribution of the axial velocity. The results obtained show that the type of the initial profile considerably affects the jet behaviour especially in the regions near the nozzle. A comparison with certain approximate methods shows good agreement between the results concerning the jet contraction and considerable differences with regard to the velocity distributions.

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